Worcester County Mathematics League

Varsity Meet 1 – October 6, 2010

Round 1: Arithmetic

1

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. Simplify: $\sqrt{6\left(2+\dfrac{1}{3}\right)\left(3+\dfrac{1}{2}\right)}$

2. Simplify: $\left(\left(\left((1+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1\right)^{-1}$

3. Multiply: $6,964\times 2,458$

ANSWERS

(1 pt.) 1.________________________________

(2 pts.) 2.________________________________

(3 pts.) 3.________________________________

Worcester Academy, Hudson, QSC & Russ Harris

Worcester County Mathematics League

Varsity Meet 1 – October 6, 2010
Round 2: Algebra 1 - Open

2

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. Steve ate 100 peanut butter cups in 5 days. Each day he ate six more than the day before. How many peanut butter cups did Steve eat on the first day?

2. At Guido's family reunion, one-third of the female guests did not speak Italian. However, half of the male guests spoke Italian. If there were three-quarters as many women as men who attended, find the ratio of guests who spoke Italian to the total number of guests. Please express your answer as a fraction reduced to lowest terms.

3. Al is able to walk 25 miles in 25 minutes less time than Bob. Al and Bob walk at constant rates and Bob walks ¼ mile per hour slower than Al. Find the speed at which Bob walks (in miles per hour).

ANSWERS

(1 pt.) 1.________________________ peanut butter cups

(2 pts.) 2.________________________

(3 pts.) 3.________________________mph

Bromfield, Algonquin, Doherty

Worcester County Mathematics League

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Round 3: Set Theory

3

All answers must be placed in the answer section at the bottom

NOTE: $\overline{S}$ **indicates the complement of the set** S

NO CALCULATOR ALLOWED

1. If set $R \neq$ set S, which of the following sets A through E is not equal to any of the others? Please use the letter name of the set for your answer.

$A = (R \cup S) \cup (R \cup S)$ $\quad$ $B = (R \cup S) \cap (R \cup S)$

$C = R \cup [S \cap (R \cup S)]$ $\quad$ $D = R \cap [S \cup (R \cup S)]$

$E = (R \cap S) \cup (R \cup S)$

2. For this problem, the universal set is $U = \{\text{positive integers less than 31}\}$. Let $A = \{\text{positive multiples of 3 less than 31}\}$, $B = \{\text{positive multiples of 4 less than 31}\}$, and $C = \{\text{positive multiples of 5 less than 31}\}$. How many elements are contained in the set $\overline{A} \cap \overline{B} \cap \overline{C}$?

3. One hundred and three students competed in the first round of the last meet of WoCoMaL last year. Seven students earned a perfect score, 4 students earned 5 points, 10 students scored 4 points, 19 students got only question 3 wrong, 61 students got question #1 correct, 44 students got question #2 correct, and 26 students got #3 correct. How many students scored zero points for the round?

ANSWERS

(1 pt.) 1. ________________________

(2 pts.) 2. ________________________

(3 pts.) 3. ________________________ students

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Round 4: Measurement

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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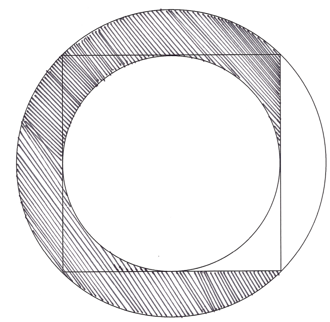
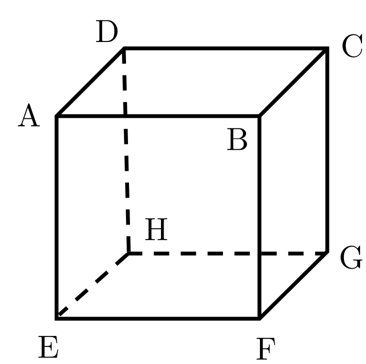
1. $MATH$ is an isosceles trapezoid with bases MA and HT. If $AT = 5 \cdot MA$, $TH = 7 \cdot MA$ and the perimeter of $MATH$ is 108, compute the area of $MATH$.



2. In the diagram, two circles are inscribed and circumscribed about a square whose side length is 8 inches. Find the area of the shaded region in square inches Please leave your answer in terms of π.

3. From cube ABCDEFGH, pyramids DACH, BACF, GCFH, and EAFH are removed. If the edge of the original cube is 9 centimeters, how many cubic centimeters of volume remain after the pyramids are removed?



ANSWERS

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________ in^3

(3 pts.) 3.__________________________ cm^3

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Round 5: Polynomial Equations

5

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. What is the only positive integral solution to the equation $n^2 + (n+1)^2 + (n+2)^2 = (n+3)^2 + (n+4)^2$?

2. Consider the quadratic equation $3x^2 - 2x + 4 = 0$. Find the equation whose roots are equal to the sum of the roots and product of the roots of the given equation. Please write your answer in the form $ax^2 + bx + c = 0$, where a, b, and c are relatively prime integers and $a > 0$.

3. The polynomial function $f(x) = x^3 + ax^2 + bx + c$ has the following properties:
   i) At least two of its zeros are distinct.
   ii) The sum of its zeros is twice the product of its zeros.
   iii) The sum of the squares of the zeros is three times the product of its zeros.
   iv) $f(1) = 1$

   Find the numerical value of c.

ANSWERS

(1 pt.) 1.__________________________

(2 pts.) 2.__________________________

(3 pts.) 3.__________________________

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places, unless stated otherwise (2 pts. each)

APPROVED CALCULATORS ALLOWED

1. What are the last two digits in the expansion of the number 12^{2010}?

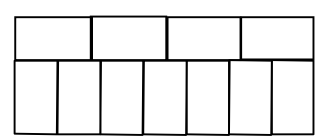
2. Let $a*b=\dfrac{a+b}{2}$, $a\Delta b=\sqrt{ab}$, and $a\otimes b=\dfrac{\frac{1}{a}+\frac{1}{b}}{2}$. Find the value of x that satisfies the equation $(9\Delta 16)*(x*4)=1.5\otimes\dfrac{3}{7}$.

3. Let $F(n+1)=\dfrac{2F(n)+1}{2}$ for all positive integers n and $F(1)=2$. Find the value of $F(2010)$.

4. It has been stated that we count in base ten because we have 10 fingers. When a Martian solves the quadratic equation $x^2-16x+41=0$ (with coefficients in his number system) he claims that the absolute value of the difference of the roots is 10 (again in his base). How many fingers does the Martian have?

5. The universal set for this problem is $U=\{1,2,3,\ldots,15\}$. If $A=\{1,7,8,11,12,13\}$, $B=\{2,3,10,11,12,14\}$ and $C=\{1,3,5,9,12,13\}$, list the elements in the set $\overline{(B\cup C)}\cup\left[\overline{A}\cap(B\cap C)\right]$. Note that $\overline{S}$ denotes the complement of set S.

6. The big rectangle in the diagram has a perimeter of 78. As you can see the rectangle is composed of 11 congruent rectangles. The total sum of all of the perimeters of those 11 rectangles is 242. Determine the area of exactly one-half of the big rectangle.



7. Circles with centers O and P have radii of length 4 and 5, respectively, and $OP=6$. The two circles intersect at points A and B. The line through A parallel to line OP intersects circle O at X and circle P at Y. Let lines XO and YP intersect at point Z. Compute the area of triangle XYZ in simplest radical form.

8. Suppose $f(x)=ax+b$ and $g(x)=bx+a$, where a and b are integers. If $f(1)=8$ and $f(g(50))-g(f(50))=28$, compute the numerical value of the product ab.

9. Let x and y be complex numbers having the following three properties:

   i) $x+y$ is a real number

   ii) $x^2+y^2=7$, and

   iii) $x^3+y^3=10$

   Under these conditions, find the maximum possible value of $x+y$.

Auburn, Worcester Academy, Hudson, St. John's, Northbridge, Doherty, Ofir Nachum, QSC (8 and 9)

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ANSWERS

Round 1

1. 7

2. $\frac{5}{8} = 0.625$

3. 17,117,512 (the commas are *not* needed)

Round 2

1. 8

2. $\frac{4}{7}$ (only)

3. $\frac{15}{4} = 3\frac{3}{4} = 3.75$

Round 3

1. D

2. 12

3. 19

Round 4

1. 576

2. 12π (only)

3. 243

Round 5

1. 10

2. $9x^2 - 18x + 8 = 0$ (yes, = 0 is needed!)

3. $-\frac{9}{4} = -2\frac{1}{4} = -2.25$

Team Round

1. 24

2. -22

3. $\frac{2013}{2} = 1006\frac{1}{2} = 1006.5$

4. 8

5. 3, 4, 6, 7, 8, 15
   or $\{3, 4, 6, 7, 8, 15\}$
   need all of the elements in any order

6. 154

7. $15\sqrt{7}$ (only)

8. 12

9. 4